Counterexamples to the Gel'fand-Kirillov Conjecture (d'après J. Alev, A. Ooms and M. Van den Bergh)

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These are notes of a talk in the UIA-algebra seminar on the paper "A class of counter examples to the Gel'fand-Kirillov conjecture" by Jacques Alev, Alfons Ooms and Michel Van den Bergh [1]. In order to outline the key ideas to ringtheorists, we restrict to the case of the non-special group PGL_n and the invariant-theoretic setting of generic matrices. Some effort was made to include proofs of basic facts on generic matrices.

1 The strategy

Let $\mathfrak{g}$ be a Lie algebra over $\mathbb{C}$ and $U(\mathfrak{g})$ (resp. $D(\mathfrak{g})$) its enveloping algebra (resp. the division ring of fractions).

Gel'fand-Kirillov conjecture : For a $\mathbb{C}$-Lie algebra $\mathfrak{g}$, $D(\mathfrak{g}) \simeq D_k(L)$ a Weyl-skewfield with center L, a purely trancendental field over $\mathbb{C}$.

Definitie 1 *Let $\mathfrak{g}$ be a Lie algebra over $\mathbb{C}$ and F the center of the division ring of fractions $D(\mathfrak{g})$. A division algebra Δ, finite dimensional over its center F is called* **$\mathfrak{g}$-bad** *iff there exists a field extension $F \subset F'$ satisfying the following properties :*

1. *The extended algebra $\Delta \otimes_F F'$ is not a domain.*
2. *There is an embedding $F \subset F' \subset D(\mathfrak{g})$.*

Theorem 1 *If $\mathfrak{g}$ is a Lie algebra admitting a $\mathfrak{g}$-bad division algebra, then $\mathfrak{g}$ is a counterexample to the Gel'fand-Kirillov conjecture.*

1.1 A filtered argument

Let $\mathbb{C} \subset F$ any field and consider the k-th Weyl algebra $A_k(F)$ with center F. This is the algebra generated by x_i, y_j, $1 \leq i \leq k$ with commutation relations

$$[x_i, x_j] = [y_i, y_j] = 0 \text{ and } [x_i, y_j] = \delta_{ij}$$

If we put $deg(x_i) = deg(y_j) = 1$, $A_k(F)$ is a filtered algebra with associated graded ring

$$gr(A_k(F)) = F[x_1, ..., x_k, y_1, ..., y_k]$$

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For $d \in A_k(F)_i - A_k(F)_{i-1}$ we denote its image in $gr(A_k(F))_i$ by $\sigma(d)$. Because $gr(A_k(F))$ is a domain, σ is multiplicative.

Let $D_k(F)$ denote its quotient ring of fractions which is a division algebra with center F. We can extend the filtration on $A_k(F)$ to a $\mathbb{Z}$-filtration on $D_k(F)$ by defining the degree and symbol of a fraction $deg(fg^{-1}) = deg(f) - deg(g)$ and $\sigma(fg^{-1}) = \frac{\sigma(f)}{\sigma(g)}$. Again, the fact that $gr(A_k(F))$ is a commutative domain makes these definitions well-defined and shows that

$$gr(D_k(F)) = Q_{gr}(F[x_1, ..., x_k, y_1, ..., y_k])$$

the $\mathbb{Z}$-graded ring obtained by inverting all homogeneous elements of $gr(A_k(F))$. Its part of degree zero is a field L, in fact it is a purely trancendental field extension of F in $2k-1$ variables, for example $\{\frac{x_2}{x_1}, ..., \frac{x_k}{x_1}, \frac{y_1}{x_1}, ..., \frac{y_k}{x_1}\}$. Further, it is then clear that the part of degree i of this graded localization is then Lx_1^i. Hence,

$$gr(D_n(F)) = F(\frac{x_2}{x_1}, ..., \frac{y_k}{x_1})[x_1, x_1^{-1}]$$

Lemma 1 *The filtration degree zero part of $D_n(F)$, D_0 is a discrete valuation ring with maximal ideal D_{-1} and residue field $F(\frac{x_2}{x_1}, ..., \frac{y_k}{x_1})$.*

Proof : (compare with [5, Prop. 3.1]) The filtration-degree allows us to define a function

$$v : D_k(F) \to \mathbb{Z} \cup \{\infty\}$$

by $v(0) = \infty$ and $v(d) = -deg(d)$ for all $0 \neq d \in D_k(F)$. Using the fact that $gr(D_k(F))$ is a commutative domain one readily verifies that $v(dd') = v(d) + v(d')$ and $v(d + d') \geq \min(v(d), v(d'))$ for all $d, d' \in D_k(F)$. Hence, v is a discrete valuation, with valuation ring D_0 and maximal ideal D_{-1} and residue field $D_0/D_{-1} = gr(D_k(F))_0$ which is the required purely trancendental field. $\square$

1.2 The proof of the theorem

Proof : (compare with [1, Prop. 3.1]) Assume that the statement of the conjecture holds for $\mathfrak{g}$, then there would be a $k \in \mathbb{N}$ such that

$$D(\mathfrak{g}) \simeq D_k(F)$$

Assume there is a $\mathfrak{g}$-bad division algebra Δ with center F and let $F \subset F'$ be the corresponding field extension. Consider the discrete valuation v on $D_k(F)$ considered above and restrict it to the commutative subfield F'. Then either of the following two cases occurs :

1. **the induced valuation is trivial.** Then going to the residue field gives the inclusions
$$F \subset F' \subset F(\alpha_1, ..., \alpha_{2k-1})$$
2. **the induced valuation is non-trivial.** Then, there is a discrete valuation ring R with field of fractions F' and residue field R/m with inclusions
$$F \subset R/m \subset F(\alpha_1, ..., \alpha_{2k-1})$$

In the first case we are done. For, consider the division algebra Δ with center F and tensor it with the purely trancendental field-extension $F(\alpha_1, ..., \alpha_{2k-1})$. We obtain

$$\Delta(\alpha_1, ..., \alpha_{2k-1})$$

which is still a division algebra, contradicting the fact that for the intermediate algebra we have that $\Delta \otimes_F F'$ is not a domain.

For the second case we can repeat the above argument provided we can show that $\Delta \otimes_F R/m$ is not a division algebra. Choose $0 \neq f \in \Delta \otimes_F F'$ with $f^2 = 0$. As R is a discrete valuation ring of F' with uniformizing parameter say π there is a natural number m such that $\pi^m f \in \Delta \otimes R$. Let $l \in \mathbb{Z}$ minimal with this property then $\pi^l f \neq 0$ in $\Delta \otimes R/m$ but still has square zero, finishing the proof. $\square$

2 The counter example

2.1 Linear algebra and invariant theory

With X_n we will denote the affine space of $n \times n$ matrix couples

$$X_n = M_n(\mathbb{C}) \oplus M_n(\mathbb{C})$$

The group $GL_n(\mathbb{C})$ acts on this space by simultaneous conjugation

$$g.(A,B) = (gAg^{-1}, gBg^{-1})$$

Clearly, the action of the center $\mathbb{C}^*.I_n \subset GL_n(\mathbb{C})$ is trivial, so we really have a $PGL_n(\mathbb{C})$-action.

Lemma 2 *The set U_n of couples (A,B) which generate $M_n(\mathbb{C})$ as a $\mathbb{C}$-algebra is a Zariski-open PGL_n-invariant set in X_n. Moreover, the PGL_n-stabilizer of any point in U_n is trivial.*

Proof : (compare with [8, 6.1 and 6.2]) If A and B do not generate $M_n(\mathbb{C})$, then the dimension of the space spanned by successive powers of A and B is $\leq n^2-1$ which can be expressed by the vanishing of $n^2 \times n^2$-minors involving polynomials in the coefficients of A and B. Hence this set is closed and it suffices to show that the complement is non-empty.

Let A be a diagonal matrix with distinct eigenvalues and let $C_1, ..., C_d \in M_n(\mathbb{C})$ which generate $M_n(\mathbb{C})$ as an algebra. Let $S_1, ..., S_k$ the list of subspaces of $\mathbb{C}^n$ which are left invariant by A (this list is finite since the eigenvalues are distinct). The C_i do not have a subspace which is simultaneously invariant (as they generate $M_n(\mathbb{C})$).

For every j we can therefore find an i such that C_i does not send S_j into itself and so there is a non-empty Zariski-open subset of $\mathbb{C}^k$

$$V_j = \{(a_1, ..., a_k) \in \mathbb{C}^k \mid (a_1C_1 + ... + a_kC_k)S_j \not\subset S_j\}$$

(observe that sending S_j into itself is a closed condition). Take a point $(c_1, ..., c_k) \in \cap_{j=1}^k V_j$, then A and $B = c_1C_1 + ... + c_kC_k$ do not have a common invariant subspace and hence they generate $M_n(\mathbb{C})$ as an algebra.

Now, take $g \in GL_n(\mathbb{C})$ such that g fixes $(A,B) \in U_n$, that is, g commutes with both A and B and hence with all of $M_n(\mathbb{C})$, so g is central. Hence, the PGL_n-stabilizer of (A,B) is trivial. $\square$

The coordinate ring $\mathbb{C}[X_n]$ is a polynomial ring in $2n^2$ variables

$$X = \begin{bmatrix} x_{11} & \dots & x_{1n} \\ \vdots & & \vdots \\ x_{n1} & \dots & x_{nn} \end{bmatrix} \quad Y = \begin{bmatrix} y_{11} & \dots & y_{1n} \\ \vdots & & \vdots \\ y_{n1} & \dots & y_{nn} \end{bmatrix}$$

The action of PGL_n on X_n induces an action by automorphisms on $\mathbb{C}[X_n]$. For example, if $g \in GL_n(\mathbb{C})$ then $g.x_{ij}$ is the (i,j)-entry of the matrix gXg^{-1} in

$M_n(\mathbb{C}[X_n])$. This action of PGL_n extends to the functionfield $\mathbb{C}(X_n)$. We would like to have a concrete description of the fixed field under this action $\mathbb{C}(X_n)^{PGL_n}$.

We need to recall a standard result in invariant-theory known as Rosenlicht's theorem, see [4, p. 143] or [9, §IV.2] for a proof. In our case it asserts that there is a Zariski-open PGL_n-stable subset $U \subset X_n$ such that $\mathbb{C}(X_n)^{PGL_n}$ is the subfield which separates orbits in U. Moreover, the trancendence degree of $\mathbb{C}(X_n)^{PGL_n}$ is then $dim(X_n) - max_{u \in U}(dim PGL_n.u)$. For a slightly stronger result see [4, §II.3.4].

Define the ring $\mathbb{G}_n$ of generic matrices as the subring of $M_n(\mathbb{C}[X_n])$ generated by the two matrices X and Y.

Lemma 3 *The fixed field $\mathbb{C}(X_n)^{PGL_n}$ is the subfield of $\mathbb{C}(X_n)$ generated by the coefficients of the characteristic polynomial of elements in $\mathbb{G}_n$. Moreover, $trdeg_{\mathbb{C}}(\mathbb{C}(X_n)^{PGL_n}) = n^2 + 1$.*

Proof : (compare with [2, p. 560-61]) In view of the action of PGL_n on $\mathbb{C}[X_n]$ it is clear that these coefficients are invariant functions, that is they are contained in $\mathbb{C}[X_n]^{PGL_n}$ and hence in the fixed field. In order to show that they generate $\mathbb{C}(X_n)^{PGL_n}$ it suffices by Rosenlicht's result to show that they separate distinct orbits in U_n.

So, let (A, B) and (A', B') be in U_n such that for all coefficients of characteristic polynomials $c_s(X, Y)$ of elements $s \in \mathbb{G}_n$ we have $c_s(A, B) = c_s(A', B')$. Then, we claim that these points belong to the same orbit.

Take an element $z(X, Y) \in \mathbb{G}_n$ such that $z = z(A, B)$ (and hence also $z' = z(A', B')$) is an $n \times n$ matrix with distinct eigenvalues. Then we can diagonalize z and z'. Hence, replacing (A, B) and (A', B') by points in their orbits we may assume that $z = z'$ a diagonal matrix with distinct eigenvalues (this operation already fixes a flag of subspaces of $\mathbb{C}^n$). Suitable polynomials $z_{11}, ..., z_{nn}$ of z can then be found such that

$$z_{ii}(A, B) = z_{ii}(A', B') = e_{ii}$$

where e_{ij} is the matrix with 1 at place (i, j) and zeroes elsewhere.

Further, there are elements $h_{ij} \in \mathbb{G}_n$ such that $h_{ij}(A, B) = e_{ij}$ (because A and B generate $M_n(\mathbb{C})$) and define $z_{ij} = z_{ii} h_{ij} z_{jj}$ then $z_{ij}(A, B) = e_{ij}$ and $z_{ij}(A', B')$ has at most one non-zero entry namely the (i, j) one. Because $1 = tr(z_{ij} z_{ji}(A, B)) = tr(z_{ij} z_{ji}(A', B'))$ we know that this $z_{ij}(A', B') \neq 0$.

Conjugating (A', B') by a diagonal matrix (and so going to another point in the orbit, if necessary) we may assume that $z_{1j}(A', B') = e_{1j}$ for all $1 \leq j \leq n$ (this operation fixes a basis in the flag).

But then it is easy to deduce that for all i, j we have $z_{ij}(A', B') = e_{ij}$. From this we can deduce that $(A, B) = (A', B')$. For example

$$A_{ij} = tr(z_{ii} X z_{ji}(A, B)) = tr(z_{ii} X z_{ji}(A', B')) = A'_{ij}$$

finishing the proof of the claim and the lemma. □

Because PGL_n acts as automorphisms on $\mathbb{C}(X_n)$, its Lie algebra $\mathfrak{sl}_n$ acts by derivations on $\mathbb{C}(X_n)$. Recall that $Der_{\mathbb{C}}(\mathbb{C}(X_n))$ is the $\mathbb{C}(X_n)$-vectorspace of all $\mathbb{C}$-derivations of $\mathbb{C}(X_n)$ and has dimension $2n^2$.

Lemma 4 *The natural map*

$$\mathbb{C}(X_n) \otimes_{\mathbb{C}} \mathfrak{sl}_n \to Der_{\mathbb{C}}(\mathbb{C}(X_n))$$

is injective.

Proof : (compare with [1, (2.3)]) Let $x = (A, B)$ be a point in U_n, then the orbit-map $\mu : PGL_n \to X_n$ determined by sending g to $g.x$ is injective. Hence so is the differential of the orbit-map

$$(d\mu)_e : T_e(PGL_n) = Lie(PGL_n) = \mathfrak{sl}_n \to T_x(X_n)$$

see for example [4, lemma p. 75]. This can also be seen directly as this map sends h to $([h, A], [h, B])$ using the natural identification $T_x(X_n) \simeq X_n$.

Now, assume $\sum_j f_j \otimes h_j$ is in the kernel of the natural map with all h_j $\mathbb{C}$-linearly independent elements of $\mathfrak{sl}_n$ and the f_j rational functions on X_n. By definition there is a Zariski-open set in X_n where all f_i are determined. So, we can choose a point $x \in U_n$ such that all f_j are defined in x and at least one $f_j(x) \neq 0$. But then the Lie-element $\sum_j f_j(x)h_j$ maps to zero in $T_x(X_n)$ a contradiction. □

2.2 Two division algebras with center $\mathbb{C}(X_n)^{PGL_n}$

Lemma 5 *The ring of generic matrices $\mathbb{G}_n$ is a domain.*

Proof : (compare with [3, Th. 22] and [7, Th. III.1.3]) First we claim that $\mathbb{G}_n$ is a prime ring. This follows if we can show that $\mathbb{G}_n\mathbb{C}(X_n) = M_n(\mathbb{C}(X_n))$ which is prime and a central extension of $\mathbb{G}_n$ (which implies that the intersection of a prime ideal with $\mathbb{G}_n$ is prime). In fact, we show that the n^2 elements X^iY^j $(0 \leq i, j \leq n)$ span $M_n(\mathbb{C}(X_n))$ as a $\mathbb{C}(X_n)$-vectorspace. This follows if we can show that

$$det(tr((X^iY^j)(X^lY^m))) \neq 0$$

(use the non-degeneracy of the trace). Now, consider the Ore-extension $\Lambda = \mathbb{C}(u)(v, \sigma)$ where $\sigma(u) = \zeta_n u$ with ζ_n a primitive n-th root of unity. Λ is a division algebra with of dimension n^2 over its center $\mathbb{C}(u^n, v^n)$ and a basis is given by the elements u^iv^j for $0 \leq i, j \leq n$, so

$$det(Tr((u^iv^j)(u^lv^m))) \neq 0$$

In view of the map $\mathbb{G}_n \to \Lambda$ sending X to u and Y to v the above determinant cannot vanish and hence $\mathbb{G}_n$ is prime.

Now, assume that $\mathbb{G}_n$ is not a domain, then there are $a, b \in \mathbb{G}_n$ such that $ab = 0$. As $\mathbb{G}_n$ is prime there is an $r \in \mathbb{G}_n$ such that $f(X, Y) = bra \neq 0$ but $f^2 = brabra = 0$.

Since $f(X, Y) \neq 0$ the induced regular map $f : M_n(\mathbb{C}) \oplus M_n(\mathbb{C}) \to M_n(\mathbb{C})$ is not the zero-map but then the same holds for extended maps $f \otimes L$ for any field-extension $\mathbb{C} \subset L$. However, since $f(X, Y)f(X, Y) = 0$ and Λ is a division algebra we have $f(x, y) = 0$ for all elements $x, y \in \Lambda$, but then $f \otimes \mathbb{C}(u, v) = 0$ as $\Lambda \otimes_{\mathbb{C}(u^n,v^n)} \mathbb{C}(u, v) \simeq M_n(\mathbb{C}(u, v))$, a contradiction finishing the proof. □

Lemma 6 *There is a division algebra Δ_n with center $\mathbb{C}(X_n)^{PGL_n}$ such that*

$$\Delta_n \otimes_{\mathbb{C}(X_n)^{PGL_n}} \mathbb{C}(X_n) \simeq M_n(\mathbb{C}(X_n))$$

Proof : Let $\mathbb{T}_n$ be the subalgebra of $M_n(\mathbb{C}(X_n))$ obtained by adjoining to $\mathbb{G}_n$ all coefficients of characteristic polynomials of its elements. The above argument can be repeated to give that $\mathbb{T}_n$ is a domain (because the map $\mathbb{G}_n \to \Lambda$ extends to $\mathbb{T}_n \to \Lambda$). Now invert all these coefficients (which are central) then we claim that $\Delta_n = \mathbb{T}_n.\mathbb{C}(X_n)^{PGL_n}$ (use lemma 3) is a division algebra. If $s \in \mathbb{G}_n$ with characteristic polynomial $s^n + a_1s^{n-1} + ... + a_{n-1}s + a_n$, then $s^{-1} = -a_n^{-1}(s^{n-1} + a_1s^{n-2} + ... + a_{n-1})$ belongs to Δ_n. The final statement follows from the proof of the foregoing lemma. □

Next, define another division algebra which is of infinite dimension over its center $\mathbb{C}(X_n)^{PGL_n}$. Let $\mathfrak{g}_n$ be the semi-direct product Lie algebra $M_n(\mathbb{C}) \oplus M_n(\mathbb{C}) \oplus \mathfrak{sl}_n$. The Lie-bracket is defined by the rule

$$[(A,B,h),(A',B',h')] = (hA' - A'h - h'A + Ah', hB' - B'h - h'B + Bh', hh' - h'h)$$

and consider the enveloping algebra $U(\mathfrak{g}_n)$.

Clearly, $M_n(\mathbb{C}) \oplus M_n(\mathbb{C}) \oplus 0$ is a commutative sub-Lie algebra of $\mathfrak{g}_n$ and we can invert all its elements to obtain an intermediate ring

$$U(\mathfrak{g}_n) \subset \mathbb{C}(X_n)\#_{\mathbb{C}} U(\mathfrak{sl}_n) \subset D(\mathfrak{g}_n)$$

where $D(\mathfrak{g}_n)$ is the division ring of fractions of the Noetherian domain $U(\mathfrak{g}_n)$ and the intermediate algebra is the $\mathbb{C}$-vectorspace $\mathbb{C}(X_n) \otimes_{\mathbb{C}} U(\mathfrak{sl}_n)$ with multiplication defined by

$$(f\#h)(f'\#h') = f(h.f')\#hh'$$

where $h.f$ is the action by derivations of $\mathfrak{sl}_n$ (and hence of its enveloping algebra) on $\mathbb{C}(X_n)$.

We now want to have another interpretation of this intermediate ring. Let $\mathcal{D}_n$ be the ring of differential operators of the field-extension $\mathbb{C}(X_n)^{PGL_n} \subset \mathbb{C}(X_n)$. That is, $\mathcal{D}_n$ is the subalgebra of the endomorphisms of $\mathbb{C}(X_n)$ generated by $\mathbb{C}(X_n)$ and the vectorspace $Der_{\mathbb{C}(X_n)^{PGL_n}}(\mathbb{C}(X_n))$ of $\mathbb{C}(X_n)^{PGL_n}$-derivations on $\mathbb{C}(X_n)$. Observe that this is a finite dimensional vectorspace of dimension

$$trdeg_{\mathbb{C}(X_n)^{PGL_n}}(\mathbb{C}(X_n)) = trdeg_{\mathbb{C}}(\mathbb{C}(X_n)) - trdeg_{\mathbb{C}}(\mathbb{C}(X_n)^{PGL_n}) = n^2 - 1$$

For more details on differential operators we refer to [6, Ch. 15].

Lemma 7 *There is a canonical isomorphism*

$$\mathbb{C}(X_n)\#_{\mathbb{C}} U(\mathfrak{sl}_n) \simeq \mathcal{D}_n$$

In particular, $D(\mathfrak{g}_n)$ is a division algebra with center $\mathbb{C}(X_n)^{PGL_n}$.

Proof : (compare with [1, Prop. 2.1]) Since PGL_n is a connected group, we have that

$$\mathbb{C}(X_n)^{\mathfrak{sl}_n} = \mathbb{C}(X_n)^{PGL_n}$$

where the first field is the subfield of $f \in \mathbb{C}(X_n)$ such that $h.f = 0$ for all h in the Lie-algebra of derivations. so the last statement follows from the first.

There is a canonical morphism

$$\mathbb{C}(X_n)\#_{\mathbb{C}} U(\mathfrak{sl}_n) \to \mathcal{D}_n$$

Both sides can be filtered, the left-hand by the Poincaré-Birkhoff-Witt filtration on $U(\mathfrak{sl}_n)$, the right-hand by the order of differential operators. The canonical map is filtration preserving and induces an algebra-morphism between the associated graded rings defined by the map on the degree one parts which is

$$\mathbb{C}(X_n) \otimes_{\mathbb{C}} \mathfrak{sl}_n \to Der_{\mathbb{C}(X_n)^{PGL_n}}(\mathbb{C}(X_n))$$

The first statement will follow if we show that this map is an isomorphism. Both sides are $\mathbb{C}(X_n)$-vectorspaces of dimension $n^2 - 1$ so it suffices to show injectivity which follows by composing with the canonical inclusion $Der_{\mathbb{C}(X_n)^{PGL_n}}(\mathbb{C}(X_n)) \subset Der_{\mathbb{C}}(\mathbb{C}(X_n))$ and lemma 4. $\square$

Summarizing, we have shown :

Theorem 2 *Δ_n is a $\mathfrak{g}_n$-bad division algebra. Consequently, the Lie algebra $\mathfrak{g}_n$ is a counter example to the Gel'fand-Kirillov conjecture.*

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