Generic Relative Crossed Products and Universal Division Algebras

Lieven Le Bruyn *
Geert Molenberghs **

Departement Wiskunde en Informatica, UIA

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Abstract

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* Research Assistant of the NFWO (Belgium)
** Research Fellow of the NFWO (Belgium)

Generic Relative Crossed Products and Universal Division Algebras

Lieven Le Bruyn*
Geert Molenberghs†
Departement Wiskunde en Informatica
Universitaire Instelling Antwerpen (UIA)
B-2610 Wilrijk (Belgium)

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Abstract

In this note we investigate a connection, suggested by E. Formanek in [7], between the crossed product problem of Amitsur's universal division algebras $UD(n)$ of degree n and the generic crossed products introduced by Rosset and Snider. Further, we introduce and study a relative version of generic crossed products which enables us to give a representation theoretic description of $UD(n)$.

1 Introduction

In 1980, E. Formanek [7] suggested a possible connection between the crossed-product problem for division algebras of dimension n^2 over their centers and representation theoretic properties of a certain lattice F_n.

This lattice occurs naturally in the study of the rationality problem of the center C_n of Amitsur's universal division algebra of degree n, $UD(n)$, see e.g. [6],[7] or [3] for details. Recall that $UD(n)$ is the classical division

*Research associate of the NFWO (Belgium)
†Research fellow of the NFWO (Belgium)

algebra of fractions of the ring G_n of 2 generic $n \times n$ matrices (which is an Ore domain) see e.g. [14, p.15 and p.175]. Its center can best be studied as a field of lattice invariants $C_n = k(L_n)^{S_n}$ where L_n is a free Abelian group of rank $n^2 + 1$ equipped with an action of the symmetric group S_n, which extends to an action by automorphisms on the group algebra kL_n and its field of fractions $k(L_n)$ see [6],[3] or section 4 below for more details.

Now, if one has an exact sequence of S_n-lattices $0 \rightarrow A \rightarrow B \rightarrow P \rightarrow 0$ where P is a permutation lattice (i.e. P has a $\mathbb{Z}$-basis which is permuted under the action of S_n) then $k(B_n)^{S_n}$ is purely trancendental over $k(A)^{S_n}$. For this reason, one is interested in establishing such sequences with $B = L_n$ and A of minimal rank possible. The best general result known today takes $A = F_n$, the Formanek lattice, which has rank $n^2 - 3n + 1$ (but note that for small values of n better choices have been found).

If G is a finite group of order n we can embed G into S_n (via the translations) such that $G \cap S_{n-1} = id$. So, we can restrict every S_n-lattice M to G and denote this G-lattice by $M \downarrow_G$. The essence of Formanek's observation is that for $n = 4$ and $G = V_4$ the Klein Vierergruppe, the restriction $F_4 \downarrow_{V_4}$ is a (minimal) relation lattice for V_4.

Recall that for any finite group G relation lattices are introduced as follows (see [8] or [11] for more details) : take a finite free presentation of G

$$1 \rightarrow R \rightarrow F \rightarrow G \rightarrow 1 \tag{1}$$

where F is a f.g. free group (on at least two generators), then the Abelianization of the group of relations R is called a relation lattice for G, $A_G = R_{ab} = R/[R, R]$. Note that A_G is a free Abelian group of finite rank equipped with a faithful G-action. If the number of generators for F is as minimal as possible (or equals 2 if G is cyclic), then A_G is called a minimal relation lattice. It turns out that minimal relation lattices often are independent on the particular choice of a (minimal) free presentation,cfr. [8].

Relation lattices turn up in the work of S. Rosset [12],[13] and R. Snider [15] on generic crossed products. For this reason, Formanek explains his observation as "probably a reflection of the fact that $UD(4)$ is a crossed product with group V_4", thereby suggesting that $UD(n)$ might be a crossed product with group G if(f) $F_n \downarrow_G$ is a (minimal) relation lattice.

However, we will show in this note that whenever G is a group of order n then both $L_n \downarrow_G$ and $F_n \downarrow_G$ are relation lattices for G provided G is non-cyclic of order $n \geq 4$ or cyclic of order $n \geq 5$. Therefore, by Amitsur's non crossed-product result [1] we get a lot of non crossed-product examples satisfying Formanek's phenomena. If we stress minimality of the relation lattice $F_n \downarrow_G$ we will show that G has to be V_4, thereby excluding known crossed-product situations such as $UD(12)$.

The upshot of the observation that $L_n \downarrow_G$ is a relation lattice for G is that we can embed canonically a generic crossed-product with group G in $M_{(n-1)!}(UD(n))$ in such a way that it coincides with the centralizer of the field of lattice invariants $k(L_n)^G$. In order to prove these facts we extend the Rosset-Snider construction (which we recall in section 2) to a relative setting in section 3. Section 4 contains the above mentioned embedding result whereas the final section is concerned with Formanek's observation.

2 Rosset-Snider generic crossed products

In [12] S. Rosset presented a large class of division algebras of degree n and order m in the Brauer group where $m \mid n$ and every prime dividing n divides m. Independently, R. Snider [15] used the same construction to study the problem whether the Brauer group is generated by cyclic algebras.

Let us briefly recall their construction : given any finite group G of order n, form a free presentation of it

$$1 \rightarrow R \rightarrow F \rightarrow G \rightarrow 1 \tag{2}$$

where F is a free group. If we divide out the commutator $R' = [R, R]$ we get a free Abelian extension

$$1 \rightarrow A \rightarrow E = F/R' \rightarrow G \rightarrow 1 \tag{3}$$

where $A = R/R'$ is a free Abelian group of finite rank equipped with a G-action (a G-lattice). If F is free on at least 2 elements this G-action is faithful [8, p.8]. The $\mathbb{Z}G$-lattice A is usually called a relation module of the group G, see [8]. The middle group E is then a torsion free Abelian-by finite group and hence its group algebra kE has a classical division ring of fractions $k(E)$ (which one calls a generic crossed product with group G) which has degree n [12, Th.1 and lemma 5].

Snider studied the center of $k(E)$ which coincides with the field of lattice invariants $k(A)^G$ which is the fixed field under the induced action of G on the group ring kA of the free Abelian group A, [15, p.283]. If one can show that this field is (stably) rational over the basefield k then by Bloch's result [4] one would have that $k(E)$ (and hence by its generic nature any crossed product with group G) is similar to a product of cyclic algebras in the Brauer group [15, Th.2] (provided k has enough roots of unity). Snider was able to prove rationality of a certain $k(A)^G$ provided $G = V_4$,the Klein Vierergruppe or G is dihedral.

However, free presentations of groups are not uniquely determined,so let us consider another free Abelian extension

$$1 \to A' \to E' \to G \to 1 \tag{4}$$

then by [8, Prop.2.4] $A \oplus \mathbb{Z}G^{\oplus a} \simeq A' \oplus \mathbb{Z}G^{\oplus a'}$ as $\mathbb{Z}G$-lattices . One way to prove this is to note that a sequence [3] determines an element of $H^2(G, A) = H^1(I_G, A)$ where I_G is the kernel of the augmentation morphism $\mathbb{Z}G \to \mathbb{Z}$, i.e. it determines an exact sequence of $\mathbb{Z}G$-lattices

$$0 \to A \to M \to I_G \to 0 \tag{5}$$

and as the sequence [?] comes from a free presentation one can show that M is a free $\mathbb{Z}G$-lattice. Similarly one has a sequence $0 \to A' \to M' \to I_G \to 0$ with M' a free $\mathbb{Z}G$-lattice and then by Schanuel's lemma we have that $A \oplus M' \simeq A' \oplus M$.

From this we deduce by [5, Prop.6 and lemme 8] that the respective centers $k(A)^G$ and $k(A')^G$ are stably equivalent over k (i.e. $k(A)^G(x_1, ..., x_u) \simeq k(A')^G(y_1, ..., y_v)$ for some u, v). Therefore, the property of being stably rational over k is preserved among all possible choices of generic crossed products with group G.

In order to obtain a more canonical definition of the Rosset-Snider generic crossed products one might restrict attention to minimal free resolutions of G i.e. sequences $1 \to R \to F \to G \to 1$ where the number of generators of F is minimal possible. In the Abelianization of such a presentation $0 \to A_{min} \to E_{min} \to I_G \to 0$, we call A_{min} a minimal relation module and $k(E_{min})$ a minimal generic crossed product with group G. No examples are known of groups with different (i.e. non-isomorphic as $\mathbb{Z}G$-lattice) minimal relation modules, giving that at least the centers of the minimal generic crossed products are isomorphic. See [8, Cor.5.20] for conditions s.t. all minimal relation modules must be isomorphic.

3 Relative generic crossed products

In this section we aim to extend the setting by allowing torsion in the middle group E, i.e. there is a subgroup H of G such that the sequence

$$1 \to A \to E \to G \to 1 \tag{6}$$

determines an element in the kernel of the restriction map $H^2(G, A) \to H^2(H, A)$. Fixing a subgroup H, the corresponding free group extensions are determined as follows.

Let F be a free group and let $F * H$ be the free amalgamated product of groups then a free representation of G relative to H is an exact sequence of groups

$$1 \to R \to F * H \to G \to 1 \tag{7}$$

and again we can Abelianize it by dividing out the commutator $R' = [R, R]$ to obtain a sequence

$$1 \to A \to E = (F * H)/R' \to G \to 1 \tag{8}$$

Again, A is a $\mathbb{Z}G$-lattice, E is a free Abelian by finite group,containing no finite normal subgroups and whose maximal finite subgroups are all conjugated to H [13]. Then, kE is a prime p.i. group ring with classical ring of quotients

$$k(E) \simeq M_h(\Delta) \tag{9}$$

where h is the order of H by Moody's theorem [10] (proving a conjecture of Rosset,[13]) and Δ is some division algebra with center the field of lattice invariants $k(A)^G$. At least if the relative relation module A is a faithful G-lattice. This is always the case unless $I_{G/H}$ (to be defined below) is locally cyclic and the normalizer of H in G is a proper normal subgroup, cfr. [9, Satz 1.21].

Definition 1 *With notations as above, Δ is called a generic relative crossed product with group G relative to the subgroup H.*

As in [12] we know that the degree of Δ in the Brauer group is equal to the order of the element in $H^2(G, A)$ corresponding to the sequence 8. We can compute this order :

Proposition 1 *Let* $1 \to A \to E = (F * H)/R' \to G \to 1$ *be the Abelianization of a relative free representation of* G *relative to* H*, then this sequence determines a generator in* $H^2(G, A) = \mathbb{Z}/d\mathbb{Z}$ *where* $d = \#(G/H)$*, hence any generic relative crossed product has degree* d *in the Brauer group of* $k(A)^G$*.*

Proof : Again, we can translate everything in terms of $\mathbb{Z}G$-lattices. Let $\mathbb{Z}G/H$ be the permutation $\mathbb{Z}G$-lattice on the cosets G/H and let $I_{G/H}$ be the kernel of the augmentation map $\mathbb{Z}G/H \to \mathbb{Z}$, then $Ker(H^2(G, A) \to H^2(H, A)) = H^1(I_{G/H}, A)$ and hence to any sequence 8 corresponds an exact sequence of $\mathbb{Z}G$-lattices

$$0 \to A \to M \to I_{G/H} \to 0 \tag{10}$$

and as we started off with a relative free presentation, we can show that M has to be a free $\mathbb{Z}G$-lattice by [9, Satz 1.8].

Now, $H^2(G, A) \simeq H^1(G, I_{G/H}) = \mathbb{Z}/d\mathbb{Z}$ where the last equality follows from the sequence $\mathbb{Z}(G/H)^G = \mathbb{Z}(\sum_x xH) \to \mathbb{Z}^G \to H^1(G, I_{G/H}) \to 0$ where x runs through a transversal for H in G. Let $1 \to A \to B \to G \to 1$ be an arbitrary generator of $H^2(G, A)$ then we get by relative freeness of sequence 8 a commutative diagram

$$\begin{array}{ccccccccc} 1 & \to & A & \to & E & \to & G & \to & 1 \\ & & \downarrow & & \downarrow & & \| & & \\ 1 & \to & A & \to & B & \to & G & \to & 1 \end{array} \tag{11}$$

Hence the order of 8 is also equal to d,finishing the proof. □

As before, we have to investigate the extend to which this construction is unique and generic. f we would have started from another relative free presentation

$$1 \to R_1 \to F_1 * H \to G \to 1 \tag{12}$$

then we would also obtain a sequence of $\mathbb{Z}G$-lattices $0 \to A' \to M' \to I_{G/H} \to 0$ with M' free, hence by Schanuel

$$A \oplus \mathbb{Z}G^{\oplus a} \simeq A' \oplus \mathbb{Z}G^{\oplus a'} \tag{13}$$

and therefore the centers of all (G, H)-generic relative crossed products are stably equivalent to one another.

Next, let us turn to the generic nature of these objects. Take a division algebra D of dimension d^2 over its center K (here, $d = \#(G/H)$), then D contains a maximal commutative subfield L which is separable of dimension d over K. Suppose that the splitting field F of L over K is Galois with group G s.t. $L = F^H$. Then, F is a subfield of $M_{\#H}(D)$ which in turn can be written as a crossed product represented by a group extension

$$1 \to F^* \to B \to G \to 1 \tag{14}$$

and the fact that L is a maximal subfield of D with $L = F^H$ can be interpreted as follows : consider the pullback diagram of the above sequence over H

$$\begin{array}{ccccccccc} 1 & \to & F^* & \to & B' & \to & H & \to & 1 \\ & & \| & & \downarrow & & \downarrow & & \\ 1 & \to & F^* & \to & B & \to & G & \to & 1 \end{array} \tag{15}$$

then the upper sequence splits or equivalently, B contains a subgroup isomorphic to H i.e. the sequence $1 \to F^* \to E \to G \to 1$ determines an element in $Ker(H^2(G, F^*) \to H^2(H, F^*)) = H^1(I_{G/H}, F^*)$ i.e. we have a sequence of $\mathbb{Z}G$-modules

$$0 \to F^* \to V \to I_{G/H} \to 0 \tag{16}$$

But then it follows from the sequence $0 \to A \to M \to I_{G/H} \to 0$ (coming from a relative free representation $1 \to A \to E \to G \to 1$) with M a free $\mathbb{Z}G$-module that there is a $\mathbb{Z}G$-morphism $M \to V$ s.t. the diagram below is commutative :

$$\begin{array}{ccccccccc} 0 & \to & A & \to & M & \to & I_{G/H} & \to & 0 \\ & & \downarrow & & \downarrow & & \| & & \\ 0 & \to & F^* & \to & V & \to & I_{G/H} & \to & 0 \end{array} \tag{17}$$

But translating this back to groupextensions, there is a groupmorphism $E \to B$ giving rise to an algebra morphism $kE \to M_{\#H}(D)$ i.e. we can get $M_{\#H}(D)$ as a specialization of $k(E) = M_{\#H}(\Delta)$ where Δ is the generic relative crossed product.

We can summarize the above facts is the following :

Theorem 1 *Let* $1 \to R \to F * H \to G \to 1$ *be a free representation of* G *relative to* H *and let* $1 \to A \to E \to G \to 1$ *be its Abelianization. Then,* kE *is a prime p.i.-algebra with ring of quotients* $k(E) = M_{\#H}(\Delta)$ *where*

Δ is a division algebra over $k(A)^G$ of order $\#(G/H)$ in the Brauer group. Moreover, kE is generic with respect to division algebras D over K which are relative crossed products w.r.t. (G,H), i.e. D contains a maximal separable subfield L which is the field of H-invariants of its splitting field which has Galois group G over K.

In fact, one can extend the foregoing construction in the folowing way : let P be a permutation G-lattice (i.e. P has a basis which is permuted by the action of G) and consider an exact sequence of $\mathbb{Z}G$-lattices

$$0 \to A \to P \to I_{G/H} \to 0 \qquad (18)$$

Then, this sequence determines again an element in $Ker(H^2(G,A) \to H^2(H,A)$ say $1 \to A \to E \to G \to 1$ and we can again consider the groupalgebra kE. This algebra has a similar generic property as the relative crossed products defined above. The crucial observation in the proof was the existence of a commutative diagram of $\mathbb{Z}G$-modules

$$\begin{array}{ccccccccc} 0 & \to & A & \to & P & \to & I_{G/H} & \to & 0 \\ & & \downarrow & & \downarrow & & \| & & \\ 0 & \to & F^* & \to & B & \to & I_{G/H} & \to & 0 \end{array} \qquad (19)$$

where the lower exact sequence determines a relative crossed product D. If P is a permutation lattice, such a diagram can be constructed by using the fact that $H^1(U,F^*) = 0$ for all subgroups U of G (Hilbert 90) and applying [2, ...]. From the morphism $P \to B$ we then construct again a morphism $kE \to M_{\#H}(D)$ proving the required generic property. Also, it should be noted that the centers of these more general generic relative crossed products are still stably equivalent to the previously defined ones.

4 The universal division algebras

Let $U_n = \mathbb{Z}S_n/S_{n-1} = \mathbb{Z}u_1 \oplus ...\mathbb{Z}u_n$ be the usual permutation representation of rank n of the symmetric group S_n and let $V_n = \mathbb{Z}S_n/S_{n-2} = \mathbb{Z}v_{12} \oplus ... \oplus \mathbb{Z}v_{n-1n}$ be the permutation lattice of rank $n(n-1)$ given by $\sigma(v_{ij}) = v_{\sigma(i)\sigma(j)}$ for all $\sigma \in S_n$. Then, there exists a sequence of $\mathbb{Z}S_n$-lattices

$$0 \to G_n \to V_n \to U_n \to \mathbb{Z} \to 0 \qquad (20)$$

where the rightmost map is the augmentation and the map $V_n \to U_n$ sends v_{ij} to $u_i - u_j$. G_n is the kernel which has rank $n^2 - 2n + 1$. Procesi [16] and Formanek [6] proved that the center of $UD(n)$ can be obtained as the field of lattice invariants $k(L_n)^{S_n}$ where $L_n = G_n \oplus U_n \oplus U_n$. It would be useful to have a similar representation theoretic description of $UD(n)$.

From the sequence of $\mathbb{Z}S_n$-lattices

$$0 \to L_n \to V_n \oplus U_n \oplus U_n \to I_{S_n/S_{n-1}} \to 0 \tag{21}$$

we obtain an Abelian groupextension

$$1 \to L_n \to E \to S_n \to 1 \tag{22}$$

defining an element in $Ker(H^2(S_n, L_n) \to H^2(S_{n-1}, L_n))$. Then, kE is a prime ring with ring of quotients $k(E) = M_{(n-1)!}(\Delta)$ where Δ is a division algebra of degree n having center $k(L_n)^{S_n}$ and it is easy to verify that

Proposition 2 $k(E) = M_{(n-1)!}(UD(n))$

Giving the desired desription of $UD(n)$. In fact, more is true :

Theorem 2 *Let G be a finite group of order n, then there exists a Rosset-Snider generic crossed product with group G , say $RS(G)$ embedded in kE. In particular, $RS(G)$ is the centrlizer in $k(E)$ of $k(L)^G$ where G is represented as a subgroup of S_n via its natural permutation representation on U_n.*

Proof : Let $G = \{id = u_1, ..., u_n\}$ act by translation on this set, then $G \hookrightarrow S_n$ s.t. $G \cap S_{n-1} = id$. Now, we can restrict the sequence of $\mathbb{Z}S_n$-lattices

$$0 \to L_n \to V_n \oplus U_n \oplus U_n \to I_{S_n/S_{n-1}} \to 0 \tag{23}$$

to G and observe the following facts :

1. $(V_n \oplus U_n \oplus U_n) \downarrow_G$ is a free $\mathbb{Z}G$-lattice
2. $(I_{S_n/S_{n-1}}) \downarrow_G \simeq I_G$ as $\mathbb{Z}G$-lattices
3. $(L_n) \downarrow_G \simeq (G_n) \downarrow_G \oplus \mathbb{Z}G \oplus \mathbb{Z}G$

So this sequence gives rise to an Abelian group extension

$$1 \to G_n \oplus \mathbb{Z}G \oplus \mathbb{Z}G \to F \to G \to 1 \tag{24}$$

and the division ring of fractions of kF is a Rosset-Snider generic crossed product $RS(G)$ with group G and having as its center $k(L)^G$. Clearly, $kF \hookrightarrow kE$ and $RS(G)$ can be interpreted as the centralizer of $k(L)^G$ in $k(E)$. $\square$

5 Formanek's observation

We continue to use the notation of the previous section. Note that the lattice G_n is generated (but not freely) by all elements of the form

$$v_{i_1 i_2} + v_{i_2 i_3} + \ldots + v_{i_q i_1} \tag{25}$$

where $i_1 \neq i_2 \neq \ldots \neq i_q \neq i_1$. One can define a morphism of $\mathbb{Z}S_n$-lattices $\pi : G_n \to U_n$ by

$$\pi(y_{i_1 i_2} + y_{i_2 i_3} + \ldots + y_{i_q i_1}) = u_{i_1} + u_{i_2} + \ldots + u_{i_q} \tag{26}$$

and one can check that π is surjective provided $n \geq 4$. Hence, we obtain an exact sequence

$$0 \to F_n \to G_n \to U_n \to 0 \tag{27}$$

of $\mathbb{Z}S_n$-lattices, where F_n is the Formanek lattice which has rank $n^2 - 3n + 1$.

If $n = 4$, then E. Formanek [7] observed that F_n considered as a V_4-lattice is isomorphic to a minimal relation module for V_4 and hence that $k(F_n)^{V_4}$ is the center of a minimal generic crossed product with group V_4. Formanek explains this as "probably a reflection of the fact that the universal division algebra $UD(4)$ is a crossed product with group V_4". Unfortunately,this seems to be merely "un accident de parcours" :

Theorem 3 *Let G be a non-cyclic group of order $n \geq 4$ (or cyclic of order ≥ 5). Then, $F_n \downarrow_G$ is always a relation module for G (thus, there is a generic crossed product with group G having center $k(F_n)^G$). Moreover, $F_n \downarrow_G$ is a minimal relation module if and only if $G = V_4$.*

Proof : We will always embed G into S_n s.t. $G \cap S_{n-1} = id$ where S_{n-1} is the subgroup fixing 1. Then $(U_n) \downarrow_G \simeq \mathbb{Z}G$, then the sequence defining F_n splits when restricted to G i.e.

$$G_n \downarrow_G \simeq F_n \downarrow_G \oplus \mathbb{Z}G \tag{28}$$

Further, note that $G_n \downarrow_G \simeq I_G \otimes I_G$ as $\mathbb{Z}G$-lattices and by [8, Prop.5.21] we know that

$$I_G \otimes I_G \simeq A_{min} \oplus \mathbb{Z}G^{\oplus(\#G-1-d(G))} \tag{29}$$

where A_{min} is a minimal relation module for G and $d(G)$ is the minimal number of generators of G. Therefore, we have

$$F_n \downarrow_G \oplus \mathbb{Z}G \simeq (A_{min} \oplus \mathbb{Z}G^{\oplus(\#G-2-g(G))}) \oplus \mathbb{Z}G \tag{30}$$

As G is assumed to be non-cyclic, A_{min} is a faithful G-lattice and hence we can apply Swan's cancellation theorem (e.g. [8, Th.5.17]) at least if $\#G - d(G) \geq 3$ to obtain that

$$F_n \downarrow_G \simeq A_{min} \oplus \mathbb{Z}G^{\oplus \#G-1-d(G)} \tag{31}$$

implying that $F_n \downarrow_G$ is a relation module in this case,cfr. [8]. Note that $F_n \downarrow_G$ is never a minimal relation module if $\#G - d(G) \geq 3$.

Now, $\#G - d(G) \leq 2$ iff G is cyclic of order 2 or 3 or $G = V_4$. For $G = V_4$ the statement is Formanek's observation and it is therefore the only case for which $F_n \downarrow_G$ is a minimal relation module.

The only place in the proof where we used non-cyclicity of G is to ensure that minimal relation modules are cyclic. If we define minimality of relation modules for cyclic groups to be those for which the ambient free group has two generators, faithfulness follows and we can repeat the foregoing proof for all cyclic groups of order ≥ 5. □

Note that while $UD(12)$ is a crossed product with group $V_4 \times \mathbb{Z}_3$, the above result gives us that $F_4 \downarrow_{V_4 \times \mathbb{Z}_3}$ is not a minimal relation module.

Still, it seems plausible that some representation theoretic conditions may be imposed if $UD(n)$ is a crossed product with group G. For, a generic crossed product with group G can be embedded in $M_{(n-1)!}(UD(n))$ and we have specializations $kE \to UD(n)$ and $UD(n) \to kE$. It would be interesting to compute the behaviour of various invariants of the groupalgebras (e.g. cyclic cohomology) in order to obtain restrictions on possible groups G.

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